Saving Math from Plato

A “Randian” Approach to the Foundations of Mathematics

All is not well in contemporary mathematics. Consider some representative quotes.

“Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.”

Bertrand Russell, philosopher of mathematics

“The present state of mathematics is a mockery of the hitherto deep-rooted and widely reputed truth and logical perfection of mathematics.”

Morris Kline, *Mathematics: The Loss of Certainty,* p. 6

“The hope of finding objective, infallible laws and standards has faded. The Age of Reason is gone.”

Kline *Ibid.,* p. 7

“How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?”
Albert Einstein

Platonism:

	Object	Status
Perception	Concretes in this world	Messy, semi-unreal (like shadows)
Reason	World of Forms	Perfect, the “really real reality”
Math	World of Forms (?)	Perfect, real

“The physical straight lines we draw are not straight; a physical tangent line does not really touch a circle at a point. In other words, physical objects fail to have the mathematical properties we study.”

Stanford Encyclopedia of Philosophy (stating the Platonic view that Aristotle sought to correct)

Euclid’s starting definitions

A point is that which has no parts

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A line is breadthless length

Pop Quiz

What are the entities or objects that mathematics describes?

a) objects in the world of Forms?

b) inhabitants of Mathematicsland?

c) ideas in the human mind?

d) elements abstracted out from perceptual concretes?

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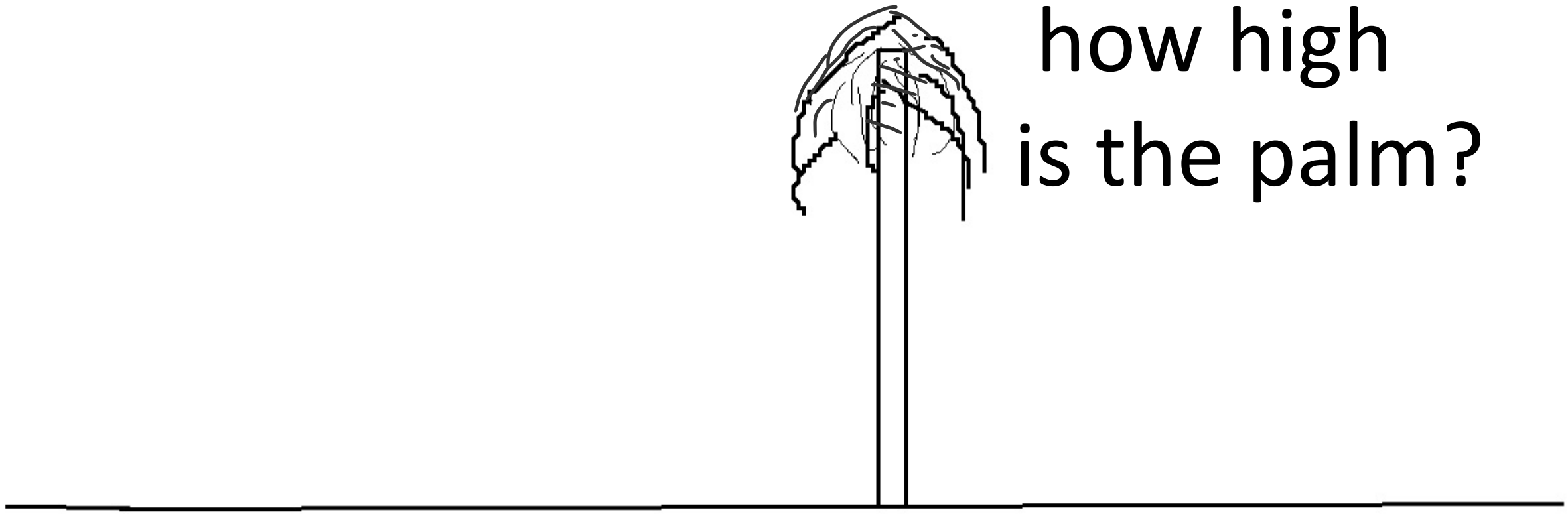
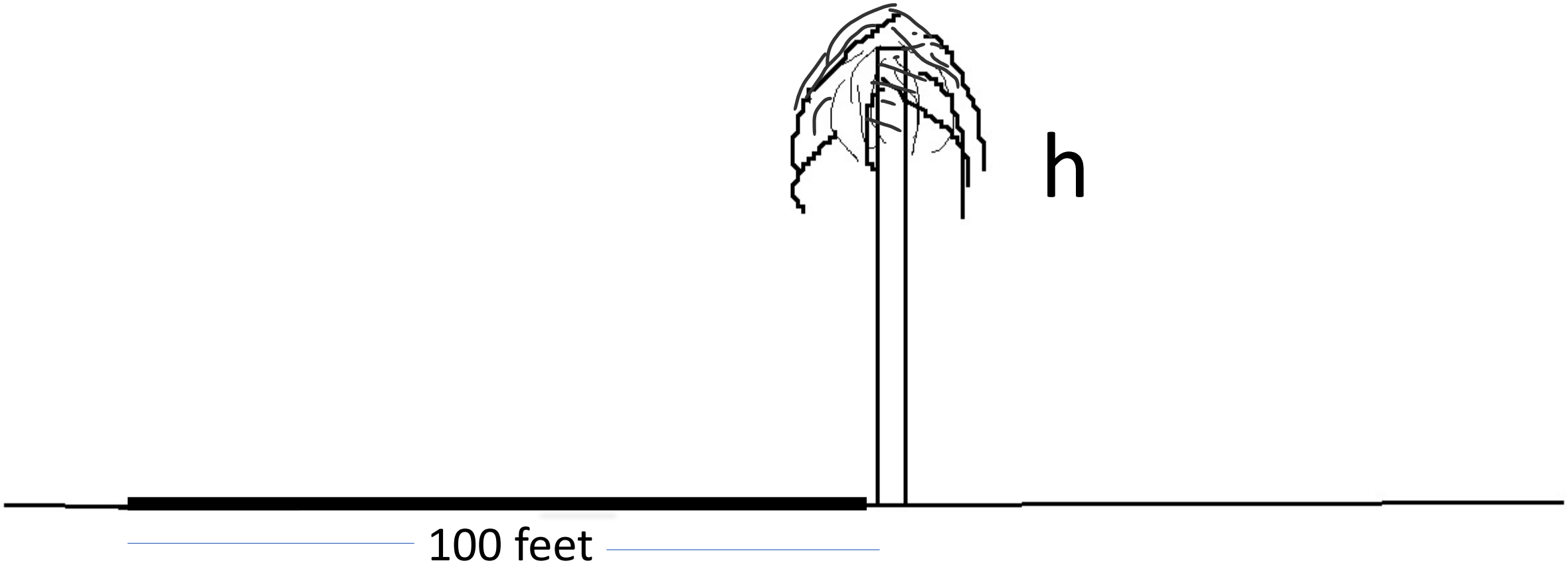
Answer: ***None of the above*** . . . as will become clear

Apply to math Ayn Rand's special question:

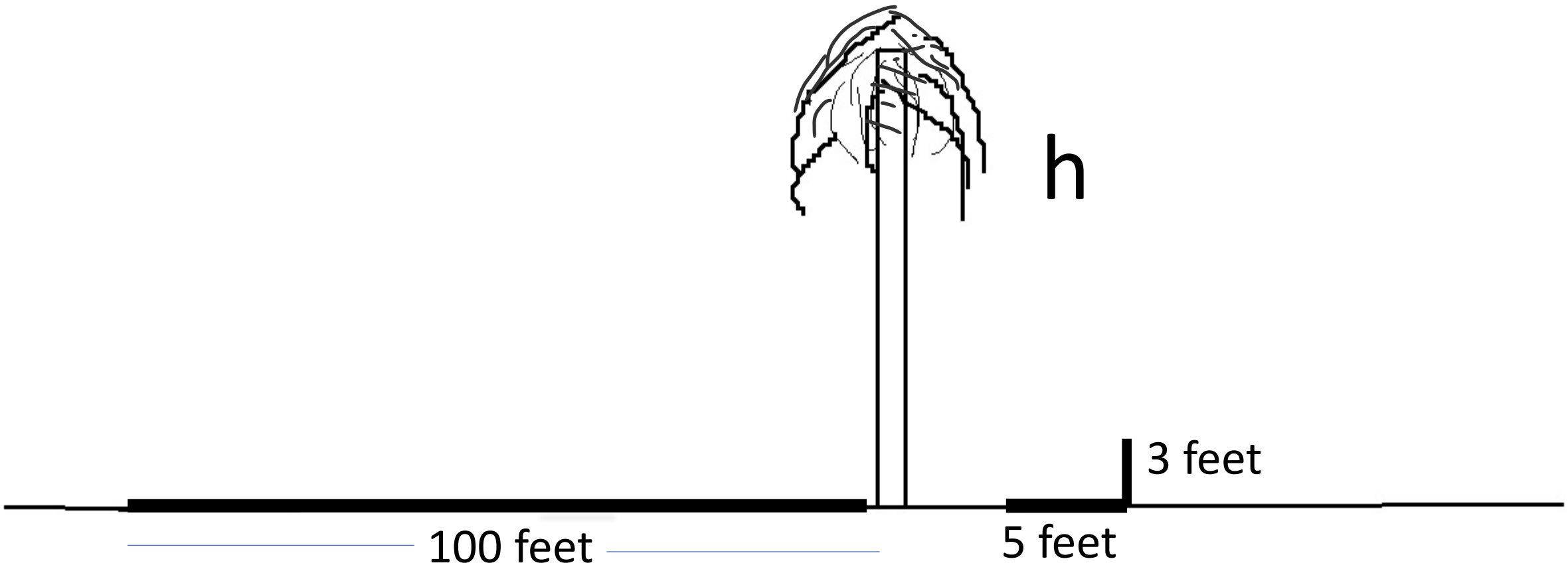
What facts of reality give rise to the need of such a thing as mathematics?



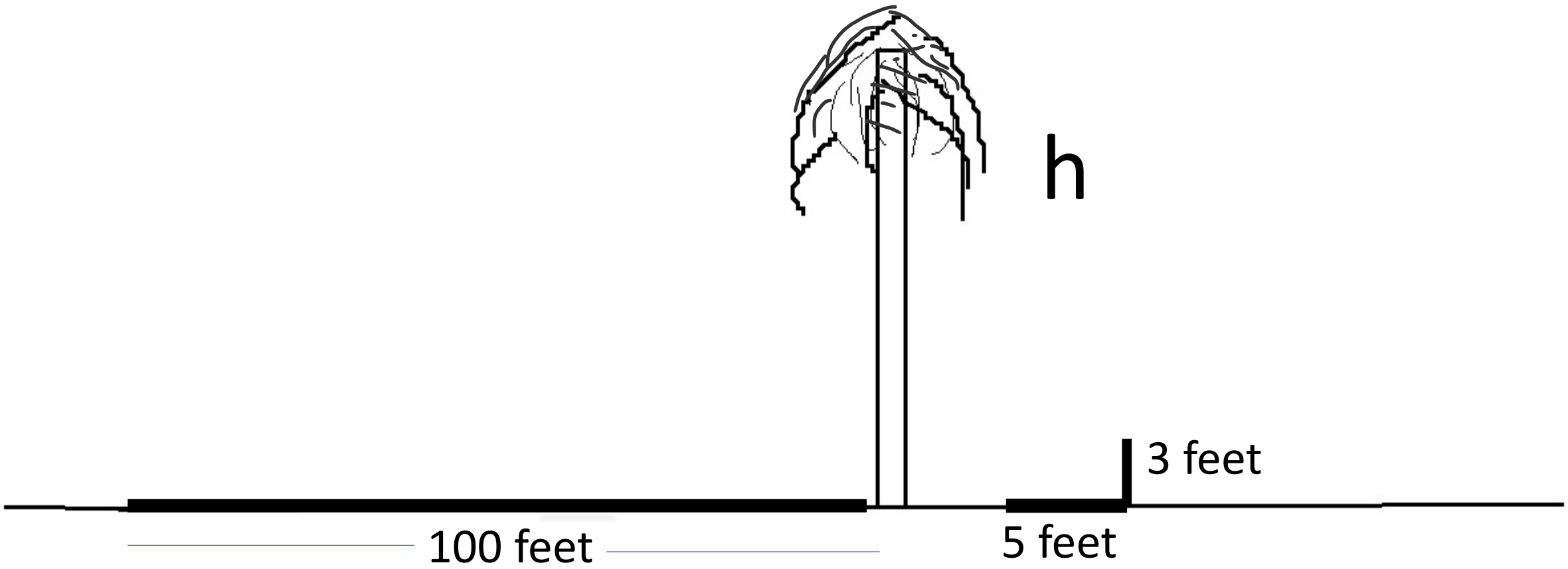
how high
is the palm?



Measure palm’s shadow



Erect a yardstick and measure its shadow



$$\frac{3}{5} = \frac{h}{100} \qquad \frac{3}{5} = \frac{60}{100}$$

h = 60 feet

So, what facts of reality give rise to the need of such a thing as mathematics? The palm-tree example indicates the answer:

> The need to acquire quantitative information—i.e., *measurements*

Defining “mathematics”

AR: “The science of measurement”

Me: The science of inferring some measurements from others—i.e., the science of calculation.

The science of calculation

-- a how-to science:

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"Mathematics is a science of *method* (the science of measurement i.e., of establishing quantitative relationships)" ITOE, Ch. 7

Math is a tool

Math is a tool for calculating measurements.

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Math does not describe things, it infers some measurements from others

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What is measurement?

"Measurement is the identification of a relationship—a quantitative relationship established by means of a standard that serves as a unit."

AR, ITOE

Quantity

An irreducible primary

Quantity: *metaphysical*

Numbers: *epistemological*

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Quantity: *metaphysical*
Numbers: *epistemological*
Numerals: *linguistic*

Quantity: *fact*

(e.g., you have so many hands)

Numbers: *concepts*

(e.g., “two”)

Numerals: *words*

(e.g., “zwei” or “2” or “II”)

Numbers measure quantity.

---How?

Two kinds of quantity

Multiplicity vs. magnitude

Two kinds of quantity

Multiplicity vs. magnitude

Entities

In a group

Discrete items

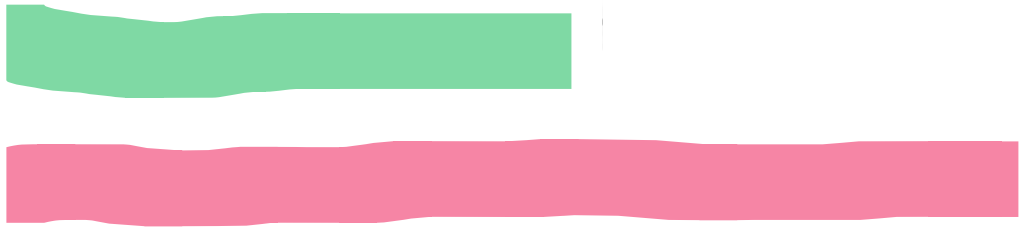
Attribute of one entity

Continuously varying

Multiplicity: two different-sized groups

X X X X X Y Y Y

Magnitudes: two different amounts of length (an attribute)



English marks the difference:

"Few" and "many": discrete multiplicity

"Little" and "much": continuous magnitude

Measuring multiplicity

Numbers measure quantity.

---How?

By a standard— “a concretely specified unit”

Measuring multiplicity

Fingers-many balls

the group of fingers is the standard of measurement

"1"

"2"

"3"

"4"

"4"

Definition of “4”: the number after 3

Definition of “3”: the number after 2

Definition of “2”: the number after 1

"4"

Definition of “4”: the number after 3
Definition of “3”: the number after 2
Definition of “2”: the number after 1
Thus, “4” entails: “1-2-3-4”— as many numerals as the balls’ quantity.

“1-2-3-4”—*the standard of measurement* for all groups of 4.

“Counting” is a process of measuring the quantity of items in a specified group by pairing a selected item with the number for unit (1) and proceeding, in strict order, through the successive numbers until no item is unpaired.

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The last number integrates into one mental unit all the symbols preceding it.

What does all this presuppose?

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Sense perception!

Contra Plato, math begins with and reduces to perception:

One vs. many:

Unit

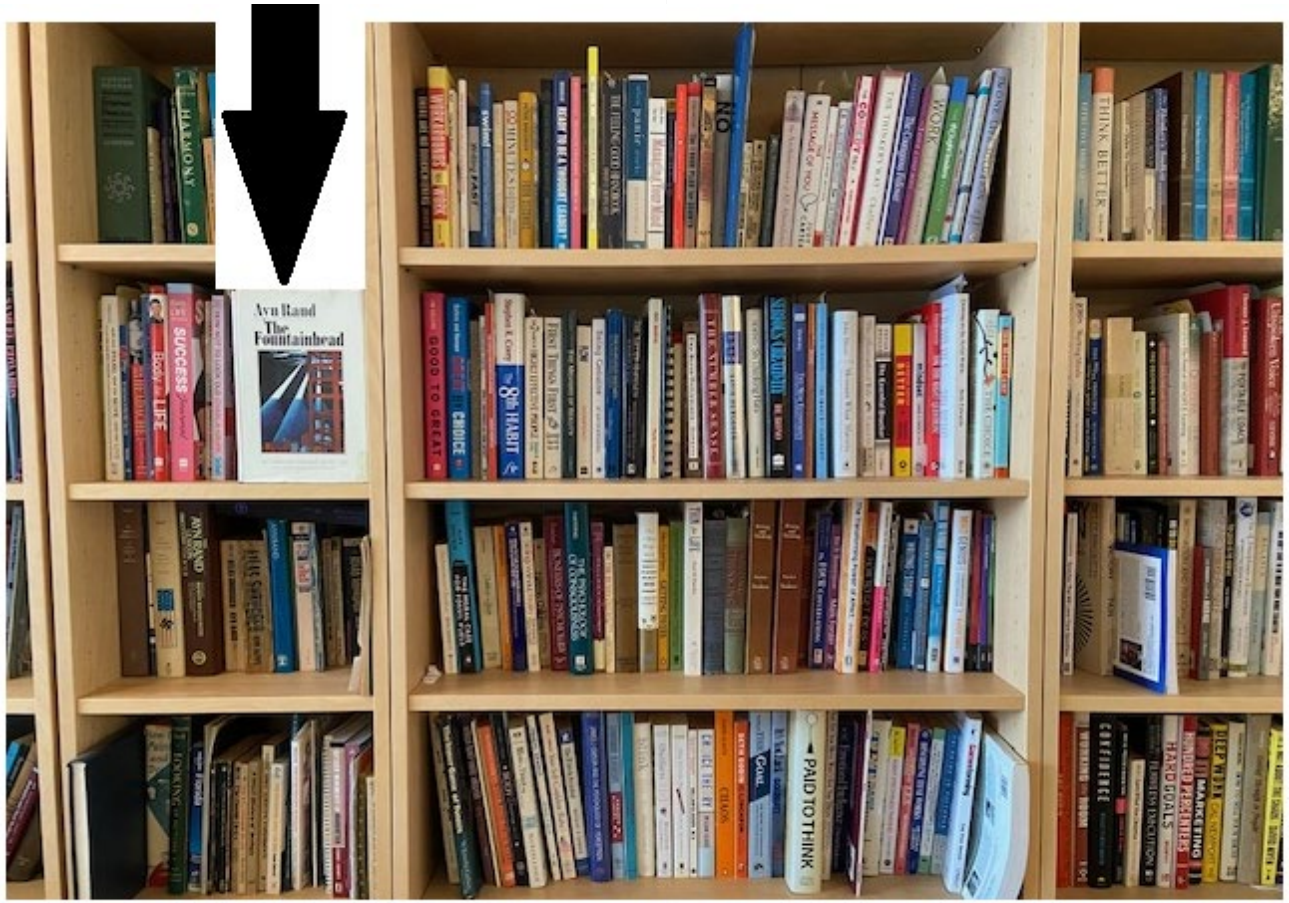
“A ‘unit’ is an existent regarded as a separate member of a group of two or more similar members.”

Perceptual bases of counting

Perception of one vs. many

“ ‘One’ is an object of perception considered apart.”
— Ayn Rand according to Allan Gotthelf

Here’s one unit!



Perceptual bases of counting

Perception of one vs. many

Perception of the numerals 1 2 3 4 5 …

Perceptual bases of counting

Perception of one vs. many

Perception of the numerals

And, indirectly, perception of . . . all the information about the world needed to count, add, etc.

How it’s done according to set theory

{} = 0

{0} = 1

{0,1} = 2

{0,1,2} = 3

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STOLEN CONCEPT!

A “number” is a member of a series of symbols in a fixed order used to measure quantity by counting or by calculation based on counting.

Hierarchy of numeric concepts:

Natural numbers (1, 2, 3, . . .)

Fractions (1/5, 2/5, 3/5, . . .)

Decimals .2, .3, .4

Zero and Negative numbers (-3, -2, -1)

For continuous magnitudes:

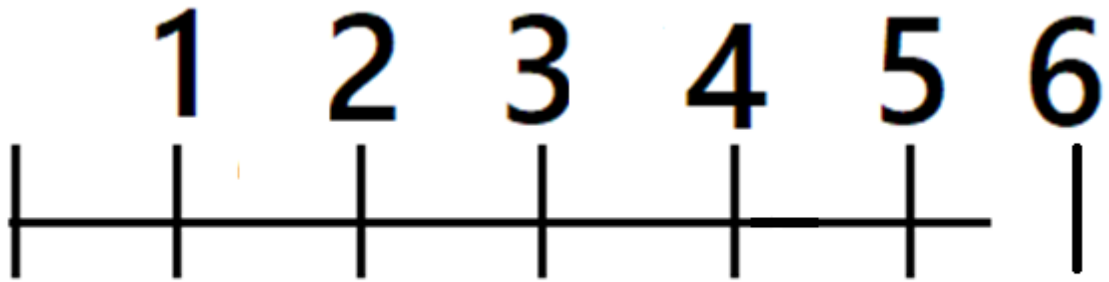
Irrationals: $\sqrt{2}$, π, e

Number line? Reals?

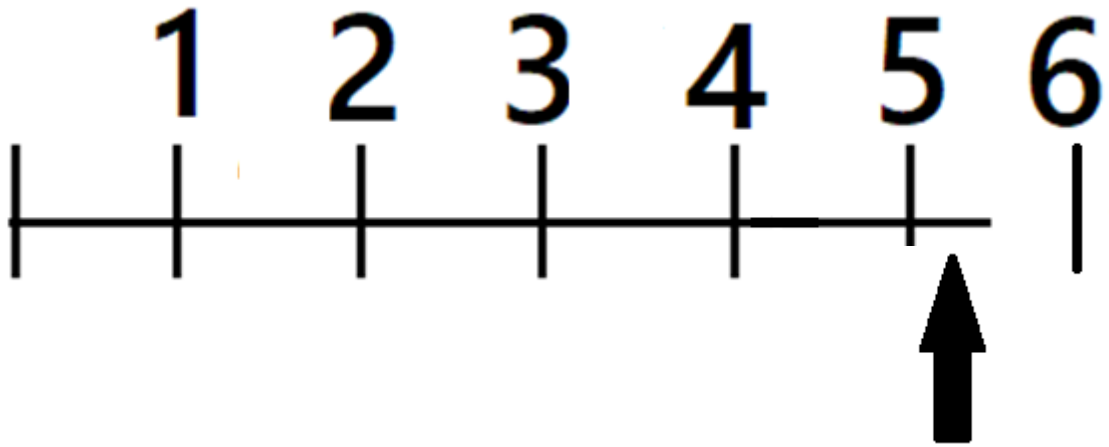
Measuring continuous magnitude: count intervals.

Measuring the continuous: counting intervals

Measuring the continuous: counting intervals



Measuring the continuous: counting intervals

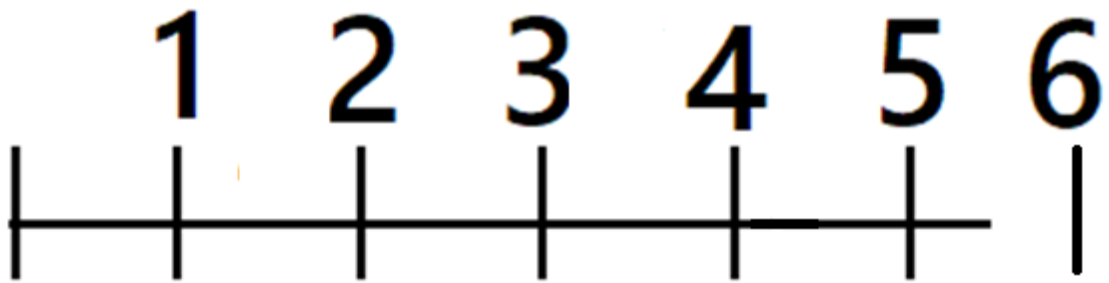


Always a bit left over

AR: “…isn't there a very simple solution to the problem of accuracy? . . .

you can always be absolutely precise simply by saying: [its length is so-and-so plus] no less than one millimeter and no more than two millimeters."

Measuring the continuous: counting intervals



Length between 5 and 6 of these intervals

Precision is contextual,
not Platonic.

Exactness, precision is contextual, not Platonic.

There is a minimum interval: ¢

Less than ¢ is “nill”: õ

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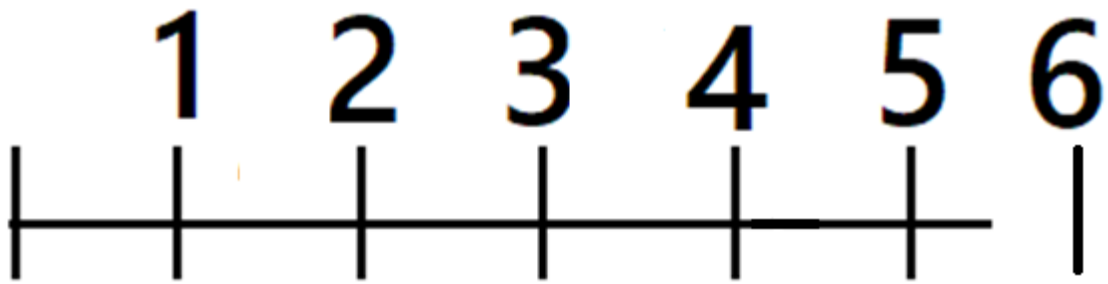
Nill is what is negligible, but not nonexistent.

Measuring the continuous: counting intervals



Length = 5 + õ

Non-Platonic geometry definitions

A volume is an entity’s extent in 3 dimensions—length, breadth, and depth

A surface is a volume of nill depth

A line is a surface of nill breadth

A point is a line of nill length.

Reality is the standard, not what is “intellectually satisfying” or “elegant.”
Mathematics has to live up to reality, not the other way ‘round.

Invalid concept:

“infinity” (∞)

Instead: “open-ended”

i.e.: iterable without *mathematical* limit

Ayn Rand on infinity:

"An arithmetical sequence extends into infinity, without implying that infinity actually exists; such extension means only that **whatever number of units does exist**, it is to be included in the same sequence." ITOE Ch. 2, p. 18 [my emphasis]

Math models reality.

Mathematics:

Almost as good as the real thing